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Error Term Propagation

1.0 PURPOSE

The purpose of this document is to provide guidance for calculating the accumulation of error terms.

2.0 SCOPE

2.1 Absolute Sum

2.1.1 This method of error propagation is known as Absolute Sum. Error is accumulated for the worst-case scenario. It is the most conservative method by which to account for error, but it includes 100% of the uncertainty and is therefore warranted where safety is a concern, such as a Class II medical device.

2.1.2 There are statistical methods to account for most of the uncertainty, and they are useful in many applications. However, they will not be discussed in this document.

3.0 DEFINITIONS

3.1 *Error Term* – Deviation of the measured value from the actual value. Also known as Uncertainty.

3.2 *Nominal Value* – The desired actual value.

3.3 *Tolerance* – A description of the uncertainty, generally in the form of the nominal value ± the error term. If a given tolerance is not symmetrical, it can be represented as symmetrical through a simple procedure described in this document.

3.4 Δz: the error term for z.

4.0 DOCUMENTATION

A device verification plan may include acceptance criteria that limits the device error. To ensure that the device error does meet that specification, the measurement error terms in the verification testing must be included in the calculation. The following example describes this concept.

Example 1:

Verify the syringe has a nominal height of 60mm ± 3mm.

One method to measure the height of the syringe is with a ruler. Say the tolerance of a specific ruler is ± 1mm. If the height of the syringe is measured to be 63mm, it seems to satisfy the specification. However, when accounting for the tolerance of the ruler, that measurement is 63mm ± 1mm. The worst-case measurement of 64mm is unacceptable. Therefore, the acceptable measured height of the syringe must be reduced by the 1mm tolerance of the ruler, to 60mm ± 2mm.

Reducing the error in the measurement will increase the acceptable measured height of the syringe. By using a set of precision calipers with a tolerance of 0.03mm, measured heights of 60mm ± 2.97mm can now be accepted. Methods for determining the accumulated error are shown below.

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4.1 Addition and Subtraction

For a calculation that involves the addition or subtraction of terms with uncertainty, the uncertainty is added to determine the overall uncertainty. This is true regardless of whether the terms themselves are added or subtracted. Here are two examples:

Example 2: Addition

x = 6.0 ± 0.2 mm,	y = 3.5 ± 0.5 mm
z = x + y	Δz = Δx + Δy
z = 6.0 + 3.5	Δz = 0.2 + 0.5
z = 9.5mm	Δz = 0.7mm

z = 9.5 ± 0.7mm

Example 3: Subtraction

x = 6.0 ± 0.2 mm,	y = 3.5 ± 0.5 mm
z = x - y	Δz = Δx + Δy
z = 6.0 – 3.5	Δz = 0.2 + 0.5
z = 2.5mm	Δz = 0.7mm

z = 2.5 ± 0.7mm

Notice that both the nominal value and the uncertainty term have the same number of significant figures. Values should be rounded to the least significant figure. For example, a measurement of 6.05 ± 0.2mm should be rounded to 6.1 ± 0.2mm. Alternatively, if the tolerance is a fabrication specification for quality assurance, it should be specified as 0.20mm.

4.2 Multiplication and Division

For multiplication of a single number with uncertainty and exact values, perform the same calculation with the uncertainty to determine that term.

Example 4:
y = 7.25 ± .50mm
A = 2y = (2)(7.25) = 14.50
ΔA = (2)(Δy) = (2)(0.50) = 1.00
A = 14.50 ± 1.00mm

For multiplication of multiple numbers with uncertainty, the equations can be derived as follows:

$$d = ab$$

$$d + \Delta d = (a + \Delta a)(b + \Delta b) = ab + a\Delta b + b\Delta a + \Delta a\Delta b$$

Since Δa is usually much less than a, and the same for Δb and b, the ΔaΔb term can be neglected. This introduces minimal error. Additionally, since *d* = *ab*, through substitution, we get:

$$\Delta d = a\Delta b + b\Delta a$$

Dividing by d, which also equals ab, we get:

$$\frac{\Delta d}{d} = \frac{\Delta a}{a} + \frac{\Delta b}{b} \qquad \text{Eqn 1}$$

Here are some examples of how to use Eqn 1:

Given:

a = 2.55 ± 0.05

b = 10.00 ± 0.50

c = 5.65 ± 0.20

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Example 5: Multiplication of Multiple Numbers

d = (a)(b)(c)

$$d = (2.55)(10.00)(5.65)$$

$$d = 144.08$$

per Eqn 1 above:

$$\frac{\Delta d}{144.08} = \frac{0.05}{2.55} + \frac{0.50}{10.00} + \frac{0.20}{5.65}$$

$$\frac{\Delta d}{144.08} = 0.11$$

$$\Delta d = 15.84$$

d = 144.08 ± 15.84

Example 6: Multiplication Using Powers

e = (a)(b) ^{2}

$$e = (2.55)(10.00)^2$$

$$e = 255.00$$

per Eqn 1 above:

$$\frac{\Delta e}{255.00} = \frac{0.05}{2.55} + \frac{0.50}{10.00} + \frac{0.50}{10.00}$$

$$\frac{\Delta e}{255.00} = 0.12$$

$$\Delta e = 30.60$$

e = 255.00 ± 30.60

For division of multiple numbers, *Eqn 1* is used in the same way:

Example 7: Multiplication of Multiple Numbers

$$d = \frac{a}{(b)(c)}$$

$$d = \frac{2.55}{(10.00)(5.65)}$$

$$d = 0.05$$

per Eqn 1 above:

$$\frac{\Delta d}{0.05} = \frac{0.05}{2.55} + \frac{0.50}{10.00} + \frac{0.20}{5.65}$$

$$\frac{\Delta d}{0.05} = 0.11$$

$$\Delta d = 0.01$$

d = 0.05 ± 0.01

5.0 REFERENCES

5.1 Lindberg, Vern. *Uncertainties and Error Propogation*. http://www.rit.edu/cos/uphysics/uncertainties/Uncertainties.html, July 1, 2000